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Plane Partitions and Their Pedestal Polynomials

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Abstract—For a linear extension P of a partially ordered set $\mathscr{S}$, we consider a generating multivariate polynomial of certain reverse partitions on $\mathscr{S}$, called P-pedestals. We establish a remarkable property of this polynomial: it does not depend on the choice of P. For $\mathscr{S}$ a Young diagram, we show that this polynomial generalizes the hook polynomial.

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1. INTRODUCTION

Let $\mathscr{S}$ be a partially ordered set. In this paper, we associate to each such set $\mathscr{S}$ a multivariate polynomial $\mathfrak{h}$. In the case where $\mathscr{S}$ is a Young diagram, the principal specialization of the polynomial $\mathfrak{h}$ coincides with the hook polynomial; see Sec. 4.

Our construction begins with the definition of the polynomial $\mathfrak{h}_P$, where P is an arbitrary linear extension of $\mathscr{S}$. Then we show that $\mathfrak{h}_P$ is, in fact, independent of P. The proof uses equality (4) (exact definitions are given in Secs. 2 and 3), which follows from a bijection between the set of reverse partitions on $\mathscr{S}$ and the product of the set of P-pedestals on $\mathscr{S}$ and the set of Young diagrams with at most $|\mathscr{S}|$ rows. To simplify the presentation, we consider the case where $\mathscr{S}$ is the set of nodes of a Young diagram λ. In this situation, linear extensions of $\mathscr{S}$ correspond to standard Young tableaux of shape λ; see Definition 1. Our results and proofs work in the same way for general $\mathscr{S}$ (and linear extensions of the set $\mathscr{S}$ instead of standard Young tableaux). After this work was completed, we were informed that the coefficients of our polynomial coincide with the values of the "β-function" in the terminology of Stanley's book [1], Theorem 3.13.1. Our approach is based on different ideas, in particular, on the factorization property (4).

2. MAIN RESULT

Let $\lambda = (\lambda_1, \dots, \lambda_l) \vdash n$, $\lambda_1 \geq \cdots \geq \lambda_l > 0$ be a partition of a number n: $\lambda_1 + \cdots + \lambda_l = n$. We identify λ with its Young diagram, i.e., with the set of nodes

$$\alpha = (i, j) \qquad \text{with} \quad j = 1, \dots, \lambda_i \quad \text{for each} \quad i = 1, \dots, l.$$

A *standard Young tableau of shape* λ is a bijection $Q\colon \lambda \to \{1, \dots, n\}$ such that the function $Q(i, j)$ increases in i and j. We denote the set of these standard Young tableaux by $\mathfrak{st}_\lambda$.

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